Name: KEY

Oscillation Problems II

1. A 150 gram mass at rest stretches a spring 6 cm when it is hanging from the spring. It is then pulled down an addition 3 cm and released.
   a. What is the spring constant?

$m = 150$ gram

$x_0 = .06$ m (equilibrium)

$A = .03$ m

$\uparrow F$

$\downarrow mg$

$\Sigma F = 0$

$F = mg$

$kx_0 = mg$

$k = \frac{(.15)(10)}{.06}$

$\boxed{k = 25 \text{ N/m}}$

   b. What is the period of the resulting oscillations?

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{.15}{25}}$

$\boxed{T = 0.49 \text{ s}}$

   c. What is the maximum speed of the mass?

$T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{0.49} = 12.9$ rad/s

$v_{max} = A\omega = (.03)(12.9)$

$\boxed{v_{max} = 0.387 \text{ m/s}}$

2. A 4 kg mass is attached to a spring with a spring constant of 350 N/m. It is oscillating with a maximum acceleration of 5 m/s^2.
   a. What is the period of the motion?

$m = 4$ kg

$k = 350$ N/m

$a_{max} = 5$ m/s^2

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4}{350}}$

$\boxed{T = 0.672 \text{ s}}$

   b. What is the amplitude of the motion?

$T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{0.672} = 9.35$ rad/s

$a_{max} = A\omega^2$

$5 = A(9.35)^2$

$\boxed{A = 0.057 \text{ m}}$

   c. What is the maximum speed of the motion?

$v_{max} = A\omega$

$= (.057)(9.35)$

$\boxed{= 0.534 \text{ m/s}}$

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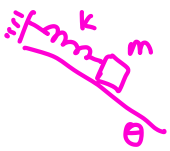
d. How much energy does the system have?

$E = K + U = K_{max} + 0 = \frac{1}{2} m v_{max}^2$

$= \frac{1}{2}(4)(.534)^2 = \boxed{0.57\ J}$

3. A 2.4 kg mass is attached to a spring on a frictionless hill with a base angle of 30°. The mass has a maximum speed of 1.5 m/s and the amplitude of the simple harmonic motion is 25 cm.
   a. What is the period of the motion?



$m = 2.4\ kg$

$\theta = 30°$

$v_{max} = 1.5\ m/s$

$A = 0.25 m$

$v_{max} = A\omega$

$1.5 = (.25)\omega$

$\omega = 6\ rad/s$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{6} = \pi/3$

$\boxed{T = 1.05\ s}$

   b. What is the spring constant?

$T = 2\pi\sqrt{\frac{m}{k}}$

$T^2 = 4\pi^2 \frac{m}{k}$

$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(2.4)}{(1.05)^2} = \boxed{86.4\ N/m}$

4. What could be the position as a function of time for a 300 gram object oscillating on the end of a spring with a spring constant of 500 N/m and a maximum speed of 2.3 m/s?

$m = 0.3\ kg$

$k = 500\ N/m$

$v_{max} = 2.3\ m/s$

Since energy is conserved

$U_{max} = K_{max}$

$\frac{1}{2}k(A)^2 = \frac{1}{2} m v_{max}^2$

$\frac{1}{2}(500)A^2 = \frac{1}{2}(.3)(2.3)^2$

$A = 0.056\ m$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{500}{.3}} = 40.8\ rad/s$

So $x = A\cos(\omega t)$ ← or sine!

$\boxed{x = (.056)\cos(40.8\ t)}$

5. A mass oscillating on a spring has a total energy of 5 J, a maximum acceleration of 12 m/s² and a frequency of 3 Hz. What is the mass?

$E = 5\ J$

$a_{max} = 12\ m/s^2$

$f = 3\ Hz$

$\therefore T = \frac{1}{3}\ s$

$T = \frac{2\pi}{\omega}$

$\omega = \frac{2\pi}{1/3} = 6\pi\ rad/s$

$a_{max} = A\omega^2$

$12 = A(6\pi)^2$

$A = 0.034\ m$

$v_{max} = A\omega = (.034)(6\pi)$

$v_{max} = 0.637\ m/s$

$K_{max} = E$

$\frac{1}{2} m(.637)^2 = 5$

$\boxed{m = 24.7\ kg}$

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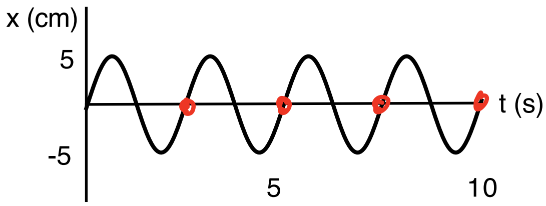
6. The position as a function of time for a 150 gram object attached to a spring is shown in the diagram below. What is the energy in the system?



4 cycles in 10 s

$T = \frac{10}{4} = \underline{2.5\ s}$

$A = .05\ m$

$m = .15\ kg$

$T = \frac{2\pi}{\omega}$ $\quad \omega = \frac{2\pi}{2.5} = \frac{4\pi}{5} = 2.51\ rad/s$

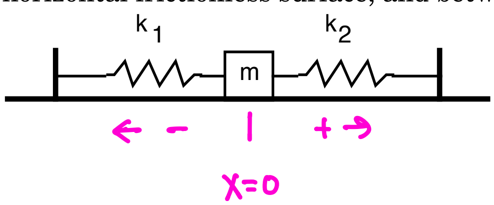
$V_{max} = A\omega = (.05)(2.51)$

$V_{max} = \underline{0.126\ m/s}$

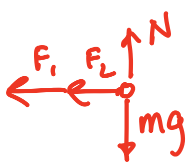
$E = K_{max} = \frac{1}{2}(.15)(.126)^2$

$\boxed{E = 0.0012\ J}$

7. Derive an expression for the period of oscillation for the system shown. The mass is on a horizontal frictionless surface, and between two springs of spring constants k_1 and k_2.



Imagine pulling m to the right, a distance x.



is the FBD

$\Sigma F_y = 0$

$F_1 =$ spring 1

$F_2 =$ spring 2

$\Sigma F_x = ma$

$-k_1 x - k_2 x = m\ddot{x}$

negatives important! if x is "+", forces are "−" and vice versa!

So $\ddot{x} = -\frac{(k_1 + k_2)}{m} x$

Hey! That is the equation for SHM because $\frac{k_1 + k_2}{m}$ is a constant

(SHM: $\ddot{x} = -\omega^2 x$ & $T = \frac{2\pi}{\omega}$)

So $\omega = \sqrt{\frac{k_1 + k_2}{m}}$!

or if you like, this equation means x is a cosine function where $\sqrt{\frac{k_1+k_2}{m}}$ is the angular frequency.

$T = \frac{2\pi}{\omega} \rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k_1 + k_2}}}$